ELSEVIER

Transient free convection of non-Newtonian fluids along a wavy vertical plate including the magnetic field effect

Cha'o-Kuang Chen, Yue-Tzu Yang, and Mong-Tung Lin
Department of Mechanical Engineering, National Cheng Kung University, Tainan, Taiwan, ROC

A Prandtl transformation method is applied to study the transient free convection of non-Newtonian fluids along a wavy vertical plate in the presence of a magnetic field. A simple transformation is proposed to transform the governing equations into the boundary-layer equations and solved numerically by the cubic spline approximation. A simple coordinate transformation is employed to transform the complex wavy surface to a vertical flat plate for a constant wall temperature by the numerical method. The effects of the magnetic field parameter, the wavy geometry and the non-Newtonian nature of the fluids on the flow characteristics and heat transfer are discussed in detail. © *1996 by Elsevier Science Inc.*

Keywords: free convection; non-Newtonian fluids; wavy vertical plate

Introduction

The free convection of a non-Newtonian fluid has been presented by many investigators because of its considerable practical applications. Because most non-Newtonian fluids are highly viscous and have a large Prandtl number, similarity solutions have been obtained for such fluids under various thermal boundary conditions (Acrivos 1960; Lee and Ames 1966; Na and Hansen 1966; Chen 1974). Kawase and Ulbrecht (1984) employed an integral method to analyze the steady-state natural convection of non-Newtonian fluids. They assumed a very thin thermal boundary layer and employed a velocity profile taken from forced-convection analysis. A review of this subject was given by Shenoy and Mashelkar (1982). They assumed a steady-state, non-Newtonian Prandtl number of unity and neglected the inertia term. Williams et al. (1987) assumed wall temperatures are a function of time and position and found nearly similar solutions for different wall temperature distribution. Nanbu (1971) estimated the limit of pure conduction for unsteady free convection on a vertical flat plate. In addition, natural convection of non-Newtonian fluids over an external surface was reported by Som and Chen (1984), Kleinstreuer et al. (1987), and Huang (1989). All previous analyses and experimental studies are available with different heating conditions for various kinds of geometries and for a variety of fluids. However, very few studies demonstrate the effects of complex geometries on such natural convection as a wavy surface, which is frequently used in finned heat exchangers and heat transfer enhancement devices. Yao (1983) proposed a single transformation to transform a complex geometry into a simple shape for which the equations of natural convection can be solved by a finite difference method. The numerical results showed that the frequency of the local heat transfer rate is twice that of the wavy surface. The transient convection heat transfer of a power-law fluid along a vertical wall was presented the first time by Haq et al. (1988). A numerical solution of the appropriate unsteady boundary-layer equations was solved numerically. The steady-state laminar natural convection heat transfer of power-law non-Newtonian fluids along a wavy vertical plate was investigated by Kim and Chen (1991). A transformation method was applied to this problem. The effects of Prandtl number, dimensionless amplitude of the wavy plate, and non-Newtonian flow index were examined in detail.

All previous analyses and experimental studies considered only flat plate or simple two-dimensional (2-D) bodies, but little has been done on non-Newtonian fluid heat transfer from a wavy surface with an imposed magnetic field. The action of a magnetic field on the fluid has many practical applications; e.g., the metals-processing industry includes the control of liquid metals in continuous casting processes, plasma welding, and the nuclear industry. Mathematical modeling of the magnetohydrodynamics problem are desirable. In the present study, the system of equations describing the transient free convection on a wavy surface is extended to a non-Newtonian fluid, including effect of the magnetic field, and solved numerically. The results of dimensionless velocity fields, temperature profiles, and heat transfer are obtained for this case. Effects of the wavy geometry and the non-Newtonian nature of the fluids on the flow and heat transfer characteristics are examined in detail.

Address reprint requests to Prof. Cha'o-Kuang Chen, Department of Mechanical Engineering, National Cheng Kung University, Tainan, Taiwan 70101, ROC.

Received 12 December; accepted 23 April 1996

Int. J. Heat and Fluid Flow 17: 604–612, 1996

655 Avenue of the Americas, New York, NY 10010

0142-727X/96/$15.00
PII S0142-727X(96)00061-9

Analysis

Electromagnetic concepts

It is well known that an electrical conductor moving in a magnetic field generates an electromotive force (emf) that is proportional to its speed of motion and the magnetic field's strength. For the coupling between the fluid flow equations and the electromagnetic fields equations to occur, the fluid must be electrically conducting, as in the case of liquid metals or gases. The field of magnetohydrodynamics is complex, because it involves the solution of both the Navier–Stokes equations characterizing fluid flow and Maxwell's equations for the magnetic field. In magnetofluidmechanics, Maxwell's equations are presented as follows:

$$\nabla \cdot B = 0 \tag{1}$$

$$\nabla \cdot D = 0 \tag{2}$$

$$\nabla \times H = J \tag{3}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{4}$$

The magnetic flux density B is expressed by

$$B = \mu_e H \tag{5}$$

$$D = \varepsilon E \tag{6}$$

where J is the current density, μ_e is the magnetic permeability, and E is the electric field intensity. By Ohm's law, the total current flow can be defined as

$$J = \sigma(E + V \times B) \tag{7}$$

where σ = electrical conductivity.

By combining the above equations, with H replaced by B/μ_e, the following induction equation is obtained:

$$\frac{\partial B}{\partial t} = \nabla \times (V \times B) + \nu_m \nabla^2 B \tag{8}$$

where $\nu_m = 1/\sigma\mu_e$

In momentum equation, we have to include the electromagnetic force, F_m, which is

$$F_m = J \times B = \sigma(V \times B) \times B \tag{9}$$

Governing equations

Consider a steady-state natural convection of non-Newtonian fluids along a wavy vertical plate imposed on a magnetic field. The physical model and coordinate system are presented in Figure 1, where (u, v) are velocity components in (x, y) directions. The surface of the plate is described by $y = \delta(x)$, where $\delta(x)$ is an arbitrary geometric function. The temperature of the plate is held at a constant value T_w, which is higher than the ambient temperature T_∞. In the present study, the electrically conducting fluids are assumed to be a non-Newtonian fluids, with (2-D) incompressible and the magnetic Reynolds number is small.

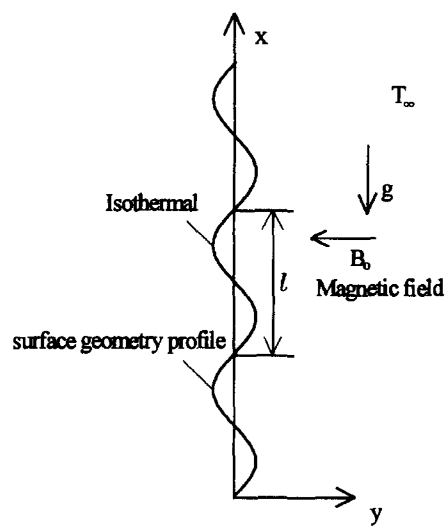


Figure 1 Physical model and coordinate

Notation

B	magnetic flux density
C_p	specific heat
P	electric displacement flux
E	electric field intensity
Ec	Eckert number
Gr	Grashof number
H	magnetic field strength
J	current density
K	thermal conductively
M_{gr}	$Mg^2 N_{gr}^{-1/2(2-n)}$
Nu	Nusselt number
N_{gr}	generalized Grashof number
P	Pressure
Pr	Prandtl number
N_{pr}	generalized Prandtl number
T	temperature
t	time
u, v	velocity components in (x, y) directions
U, V	dimensionless velocity components
x, y	coordinates
X, Y	dimensionless coordinates

Greek

α	amplitude wave
β	thermal expansion coefficient
δ	surface geometry function
θ	dimensionless temperature
μ	viscosity
μ_e	magnetic permeability
ρ	density
σ	electrical conductivity
τ	dimensionless time

Superscripts

—	dimensionless quantity
′	derivative with respect to x

Subscripts

w	wall
∞	free stream

The properties of the fluids are assumed to be constant, except for the density in the buoyancy force term. A magnetic field with a constant magnetic flux density Bo is applied. In magnetofluid-mechanics, fluid motion is governed by the laws of conservation of mass, momentum, and energy. The equation of continuity remains unchanged. The momentum and energy equations must be modified from Maxwell's field equation and Ohm's law. Based on the above assumptions, the governing equations of continuity, momentum, and energy for the steady-state natural convection of non-Newtonian fluids along a wavy vertical plate, including the magnetic field effect, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{10}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}\right)+g\beta(T-T_\infty)-\frac{\sigma B_0^2}{\rho}u \tag{11a}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}-\frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial x}+\frac{\partial \tau_{yy}}{\partial y}\right) \tag{11b}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial^2 x^2}+\frac{\partial^2 T}{\partial^2 y^2}\right)+\frac{\sigma B_0^2}{\rho C_p}u^2 \tag{12}$$

Prandtl's transposition theorem

The first step is to transform the irregular wavy surface into a flat surface by use of Prandtl's transposition theorem, Yao (1988). The theorem is that the flow is displaced by the amount of the vertical displacement of an irregular solid surface, and the vertical component of the velocity is adjusted according to the slope of the surface. The form of the boundary-layer equations is invariant under the transformation, and the surface conditions can be applied on a transformed flat surface. This allows the boundary conditions to be easily incorporated into any numerical method. To transform the above governing equations, the following dimensionless quantities are introduced.

$$\bar{x}=\frac{x}{\ell} \tag{13a}$$

$$\bar{y}=\frac{y-\delta}{\ell}N_{gr}^{1/2(n+1)} \tag{13b}$$

$$\bar{u}=\frac{u}{\sqrt{\ell g\beta\Delta T}}=\frac{u}{u_\infty} \tag{13c}$$

$$\bar{v}=\frac{v-\delta' u}{\sqrt{\ell g\beta\Delta T}}N_{gr}^{1/2(n+1)}=\frac{v-\delta' u}{u_\infty}N_{gr}^{1/2(n+1)} \tag{13d}$$

$$\delta'=\frac{d\delta}{dx}=\frac{d\bar{\delta}}{dx},\qquad \bar{\delta}=\frac{\delta}{\ell} \tag{13e}$$

$$\bar{P}=\frac{P}{\rho\ell g\beta\Delta T}=\frac{P}{\rho u_\infty^2} \tag{13f}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\qquad \bar{t}=\frac{t}{\sqrt{\dfrac{\ell}{g\beta\Delta t}}}=\frac{u_\infty t}{\ell} \tag{13g}$$

$$N_{gr}=\frac{\rho^2\ell^{n+2}[g\beta\Delta T]^{2-n}}{m^2} \tag{13h}$$

$$N_{pr}=\frac{\rho C_p}{k}\left(\frac{m}{\rho}\right)^{2/(1+n)}(\ell)^{(1-n)/(1+n)}[\ell g\beta\Delta T]^{3(n-1)/2(1+n)} \tag{13i}$$

where N_{gr} and N_{pr} are the generalized Grashof number and the generalized Prandtl number, respectively. Using Equation 13b, we transform the wavy surface into a flat surface. Neglecting small order in N_{gr} the governing equations are transformed to

$$\frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{14}$$

$$\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}=-\frac{\partial \bar{P}}{\partial \bar{x}}+\delta' N_{gr}^{1/2(n+1)}\frac{\partial \bar{P}}{\partial \bar{y}}+\theta+(1+\delta'^2)\times\frac{\partial}{\partial \bar{y}}\left(\left|\frac{\partial \bar{u}}{\partial \bar{y}}\right|^{n-1}\frac{\partial \bar{u}}{\partial \bar{y}}\right)-M_g^2N^{1/2(2-n)}\bar{u} \tag{15}$$

$$\delta''\bar{u}^2+\delta'\theta=\delta'\frac{\partial \bar{P}}{\partial \bar{x}}-(1+\delta'^2)\times N_{gr}^{1/2(n+1)}\frac{\partial \bar{P}}{\partial \bar{y}}-(1+\delta'^2)\times N_{gr}^{1/2(n+1)}\frac{\partial \bar{P}}{\partial \bar{y}}+\delta' M_g^2 N_{gr}^{-1/2(2-n)}\bar{u} \tag{16}$$

$$\frac{\partial \theta}{\partial \bar{t}}+\bar{u}\frac{\partial \theta}{\partial \bar{x}}+\bar{v}\frac{\partial \theta}{\partial \bar{y}}=\frac{1}{\mathrm{Pr}}(1+\delta'^2)\frac{\partial^2\theta}{\partial \bar{y}^2}+M^2\,\mathrm{EcGr}^{-1/2}\,\bar{u}^2 \tag{17}$$

where

$$M_g^2=\frac{\sigma B_0^2\rho^{1/(2-n)}\ell^{2/(2-n)}}{\rho m^{1/(2-n)}},\qquad M^2=\frac{\sigma B_0^2\ell^2}{\rho\nu} \tag{18}$$

The transformed momentum Equation 15 and 16 can be combined into one equation by neglecting the pressure gradient. The resulting dimensionless governing equations are:

$$\frac{n+1}{n}U+[2(n+1)X]\frac{\partial U}{\partial X}-Y\frac{\partial U}{\partial Y}+[2(n+1)X]^{(n-1)(2n+1)/2n(n+1)}\frac{\partial V}{\partial Y}=0 \tag{19}$$

$$\frac{\partial U}{\partial \tau} + [2(n+1)X]^{1/n} U \frac{\partial U}{\partial X} + \{[2(n+1)X]^{(1-n)/2n(n+1)} V - [2(n+1)X]^{(1-n)/n} UY\} \frac{\partial U}{\partial Y} = (1+\delta'^2)^{-1}\left\{\theta - M_g^2 N_{gr}^{-1/2(2-n)} [2(n+1)X]^{1/n}\right\} U^2 - \left\{\frac{n+1}{n}[2(n+1)X]^{(1-n)/n} + \frac{1}{(1+\delta^2)}\delta'\delta''[2(n+1)X]^{1/n}\right\} U^2 + (1+\delta'^2)\frac{\partial}{\partial Y}\left(\left|\frac{\partial U}{\partial Y}\right|^{n-1}\frac{\partial U}{\partial Y}\right) \tag{20}$$

$$\times [2(n+1)X]^{(n-1)/2n(n+1)} \frac{\partial \theta}{\partial \tau} + [2(n+1)X]^{(3n+1)/2n(n+1)} U \frac{\partial \theta}{\partial X} + \{V - [2(n+1)X]^{(1+2n)(1-n)/2n(n+1)} UY\}\frac{\partial \theta}{\partial Y} = \Pr^{-1}(1+\delta'^2)\frac{\partial^2 \theta}{\partial Y^2} + M^2\,\mathrm{Ec}\mathrm{Gr}^{-1/2}[2(n+1)X]^{(2n+1)/n(n+1)} U^2 \tag{21}$$

Table 1 Function of F, G, and S

F_{ij}^{θ}	$\theta_{ij}^k - [2(n+1)X_i]^{1/n} U_{ij}^k \dfrac{\theta_{ij}^k - \theta_{i-1,j}^k}{X_i - X_{i-1}} \Delta\tau$
G_{ij}	$\left\{[2(n+1)X_i]^{(1-n)/n} U_{ij}^k Y_j - \dfrac{V_{ij}^k}{[2(n+1)X_i]^{(n-1)/2n(n+1)}}\right\}\Delta\tau$
S_{ij}	$\dfrac{1+\delta'^2}{\Pr}\,\dfrac{\Delta\tau}{[2(n+1)X_i]^{(n-1)/2n(n+1)}}$
F_{ij}^{U}	$U_{ij}^k - [2(n+1)X_i]^{(1-n)/n} Y_j U_{ij}^k mU_{ij}^k \Delta\tau + \dfrac{\Delta\tau}{1+\delta_i'^2}\{\theta_{ij}^k - Mg^2 N_{gr}^{-1/2(2-n)}[2(n+1)X_i]^{1/2n} U_{ij}^k - U_{ij}^{k2}\left\{\dfrac{n+1}{n}[2(n+1)X_i]^{(1-n)/n} + \dfrac{\delta_i'\delta_i''}{1+\delta_i'^2}[2(n+1)X_i]^{1/n}\right\}\Delta\tau - [2(n+1)X_i]^{1/n} U_{ij}^k \dfrac{U_{ij}^k - U_{i-1,j}^k}{X_i - X_{i-1}}\Delta\tau\Big\}/\{1 - [2(n+1)X_i]^{(1-n)/n} Y_j mU_{ij}^k \Delta\tau\}$
G_{ij}^{U}	$\Big\{[2(n+1)x_i]^{\frac{1-n}{n}\frac{k}{U_{ij}Y_j\Delta\tau}} - [2(n+1)X_i]^{(1-n)/2^n(n+1)V_{ij}^k}\Delta\tau + (1+\delta_i'^2)\dfrac{\lvert mU_{ij}^k\rvert^{n-1} - \lvert mU_{i,j-1}^k\rvert^{n-1}}{Y_j - Y_{j-1}}\Delta\tau\Big\}/\{1 - [2(n+1)X_i]^{(1-n)/n} Y_j mU_{ij}^k \Delta\tau\}$
S_{ij}^{U}	$(1+\delta_i'^2)\lvert mU_{ij}^k\rvert^{n-1}\Delta\tau/\{1 - [2(n+1)X_i]^{(1-n)/n} Y_j mU_{ij}^k \Delta\tau\}$

Table 2 Comparison with the results of Yao (1983) for different grid number

Grid number	41*41	41*46	41*81	41*101
Local Nusselt number	0.5693	0.5672	0.5662	0.5656
Grid number	61*41	61*61	61*81	61*101
Local Nusselt number	0.5693	0.5673	0.5662	0.5657
Grid number	81*41	81*61	81*81	81*101
Local Nusselt number	0.5694	0.5673	0.5663	0.5657
Grid number	101*41	101*61	101*81	101*101
Local Nusselt number	0.5694	0.5674	0.5664	0.5658
Grid number (Yao 1983)	161*501			
Local Nusselt number	0.5671 (X=2.0); Pr=1.0; M_{gr}=0.0; α=0.0; n=1.0; X=4.0; Y=10.0			

where

$$X = \bar{x} \tag{22a}$$

$$Y = \bar{y}/[2(n+1)X]^{1/2(n+1)} \tag{22b}$$

$$U = \bar{u}/[2(n+1)X]^{1/2n} \tag{22c}$$

$$V = [2(n+1)X]^{1/(2n+1)}\bar{v} \tag{22d}$$

$$\theta = \theta, \qquad \tau = \bar{t}/[2(n+1)X]^{1/2n} \tag{22e}$$

with the corresponding boundary conditions

$$X = 0, \qquad U = \theta = 0$$

$$\text{at } Y = 0, \qquad U = V = 0, \qquad \theta = 1 \tag{23}$$

$$\text{as } Y \to \infty, \qquad U = \theta = 0 \tag{24}$$

The local Nusselt number and the averaged Nusselt number can be determined by using Newton's cooling law and Fourier's law,

$$\mathrm{Nu}_x = -[N_{gr}/2(n+1)X]^{1/2(n+1)}(1+\delta'^2)^{1/2}\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0} \tag{25}$$

$$\overline{\mathrm{Nu}_x} = -\frac{1}{S}\int_0^X [N_{gr}/2(n+1)X]^{1/2(n+1)}(1+\delta'^2)^{1/2}\left.\frac{\partial\theta}{\partial Y}\right|_{Y=0} dX \tag{26}$$

where

$$S = \int_0^X [1+\delta'^2]^{1/2}\, dx$$

Numerical analysis

The governing equations with the corresponding constant temperature boundary condition were solved by using the cubic spline approximation method (Rubin and Graves (1975). The SADI procedure was applied to perform the numerical computation. Using the spline formulation, a natural convection boundary-layer equation is written in the following form.

$$\phi_{ij}^{n+1} = F_{ij} + G_{ij}(m\phi)_{ij}^{n+1} + S_{ij}(M\phi)_{ij}^{n+1} \tag{27}$$

where

$$(m\phi)_{ij}^{n+1} = \left(\frac{\partial\phi}{\partial Y}\right)_{ij}^{n+1} \tag{28}$$

$$(M\phi)_{ij}^{n+1} = \left(\frac{\partial^2\phi}{\partial Y^2}\right)_{ij}^{n+1} \tag{29}$$

and function of F, G, and S are shown in Table 1.

In this study, the iteration process is continued until the convergence criterion, is achieved

$$\left|\frac{\phi_{ij}^{n+1} - \phi_{ij}^{n}}{\phi_{\max}^{n}}\right| < 10^{-4} \tag{30}$$

Results and discussion

The effects of the magnetic field parameter M_{gr}, the wavy geometry α, and the power-law index n on flow characteristics and heat transfer have been studied. To verify the numerical accuracy of the solution, numerical results were first obtained for the case of Newtonian fluid ($n = 1.0$) with constant wall temperature and compared to those of reported by Yao (1983), as shown in Table 2, which is a comparison of the present calculation of local Nusselt number with different grid numbers. The calculated

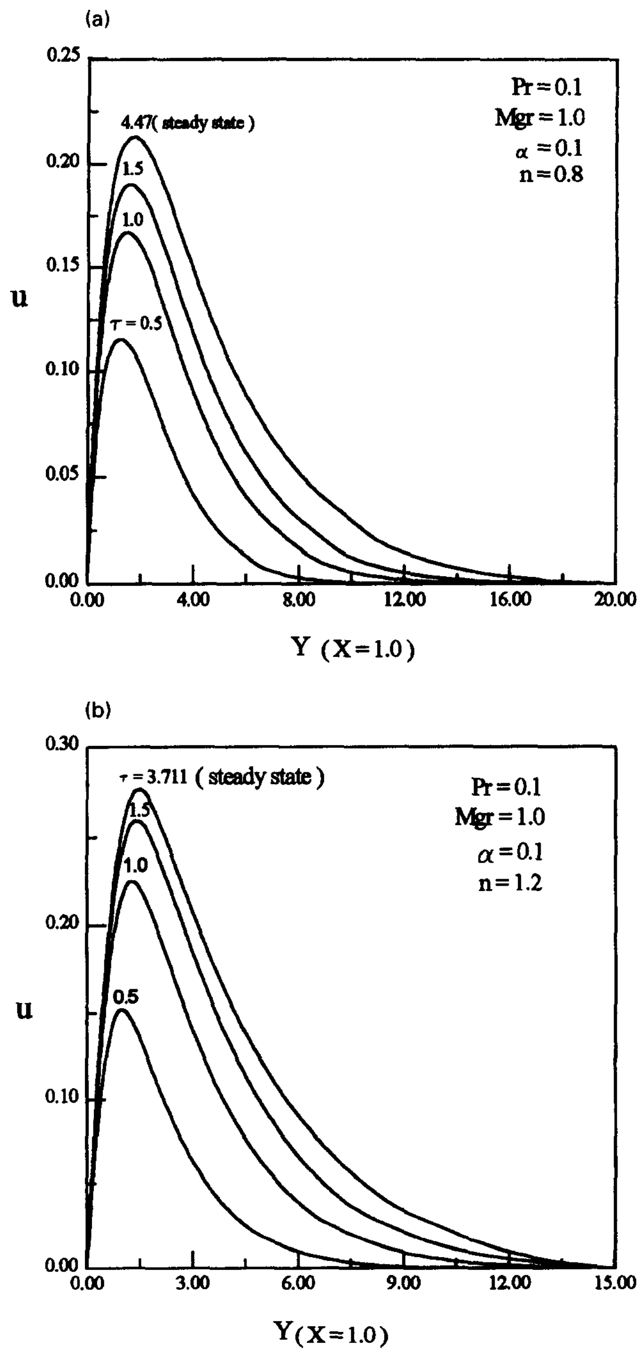


Figure 2 (a) Transient dimensionless axial velocity distribution for $\alpha=0.1$, Pr$=0.1$, and $n=0.8$; (b) transient dimensionless axial velocity distribution for $\alpha=0.1$, Pr$=0.1$, and $n=1.2$

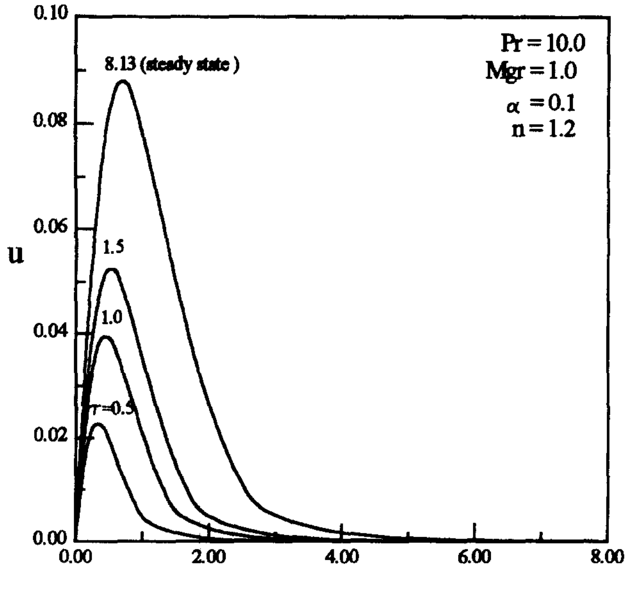
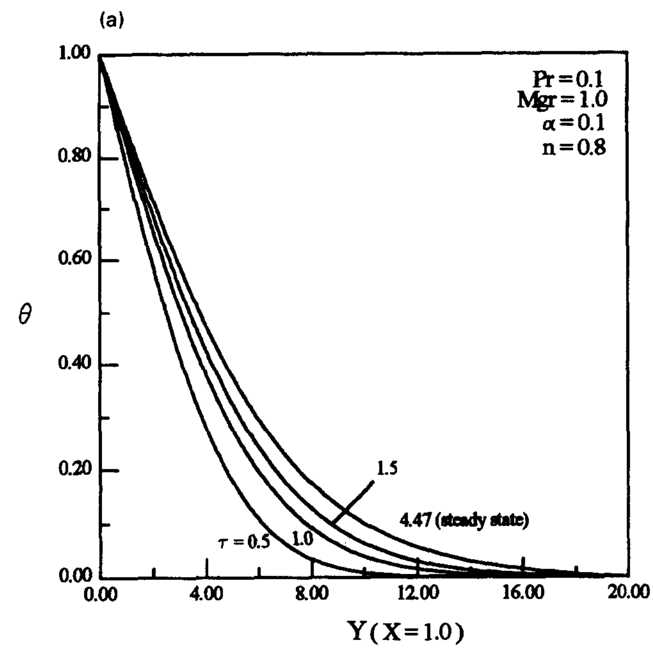


Figure 3 Transient dimensionless axial velocity distribution for $\alpha=0.1$, Pr=10, and $n=1.2$

solutions appear to be independent of grid number of X-axis. The results agree well with grid number of $y = 61$. It also demonstrates that cubic spline approximation saves much CPU time. The present calculations in the absence of magnetic field are in good agreement with the results of Yao. Therefore, the present results should have a relatively high degree of accuracy, although no available precise results can be compared with the present calculations. The numerical results are presented for $\delta = \alpha \sin(2\pi x)$ to demonstrate the advantage of the transformation method. Figures 2a and b and Figure 3 represent the transient axial velocity distribution for Pr = 0.1 and 10, $n = 1.2$ and 0.8. With the increase of n, the peak value of axial velocity increases, but the velocity boundary layer becomes thinner. The effects of n on the transient temperature distribution with Pr = 0.1 and $\alpha = 0.1$ are shown in Figure 4a and b. It demonstrates that the dilatant fluid ($n = 1.2$) has a thinner thermal boundary layer. The transient local heat transfer coefficient distributions for Pr = 0.1, $\alpha = 0.1$, $n = 0.8$, and 1.2 are shown in Figure 5a and b. It shows that the peak of the heat transfer rate after one wavelength from the leading edge is shifted slightly upstream from the trough and the crest. Downstream the heat transfer varies according to the orientation of the surface. For $\alpha = 0.1$, $n = 0.8$, and 1.2, the transient averaged heat transfer coefficient are plotted in Figure 6a and b. The averaged Nusselt number per unit wavelength is defined as

$$\overline{\mathrm{Nu}\, N_{gr}^{n}} = \frac{\int_0^{X_{max}} \overline{\mathrm{Nu}}\, N_{gr}^{n}\, dx}{\int_0^{X_{max}} \sqrt{(1+\delta'^2)}\, dx} \tag{31}$$

The total heat transfer rate for a wavy surface, considering larger heat transfer area, is about the same as that of a flat plate. The influences of wave amplitude on heat transfer characteristics are examined, as shown in Figure 7a and b. With constant magnetic strength $M_{gr} = 1.0$, Pr = 0.1, $n = 1.2$, and 0.8, increasing the wave amplitude α from 0.0 to 0.2 will decrease the averaged Nusselt number. The effects of magnetic field strength are presented in Figure 8a and b. In the case of the absence of a magnetic field, averaged Nusselt number approaches a minimum value, then it increases as time increases. The above trend is not obvious in the case of large Prandtl number. Figure 9a and b display the effect of Prandtl number on the averaged Nusselt number. In the case of large Prandtl number, the average Nusselt number per unit wave length is higher than the low Prandtl number.



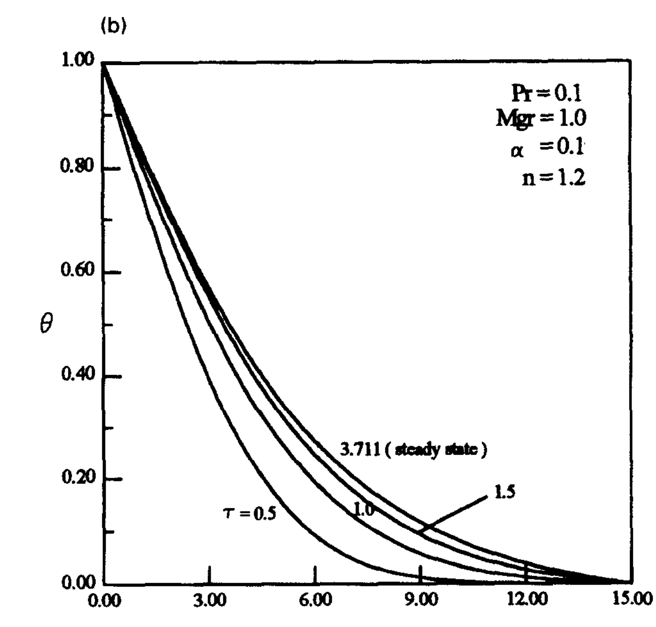


Figure 4 (a) Transient dimensionless temperature distribution for $\alpha=0.1$, Pr=0.1, and $n=0.8$; (b) transient dimensionless temperature distribution for $\alpha=0.1$, Pr=0.1, and $n=1.2$

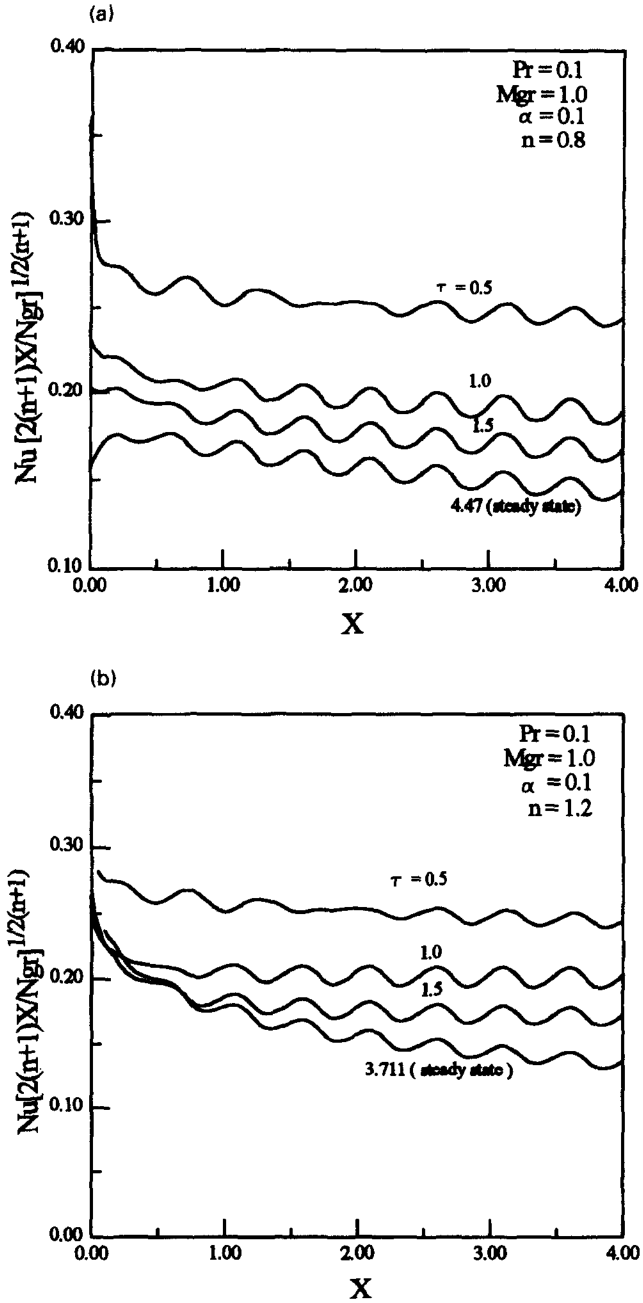


Figure 5 (a) Transient local heat transfer coefficient distribution for $\alpha=0.1$, $Pr=0.1$, and $n=0.8$; (b) transient local heat transfer coefficient distribution for $\alpha=0.1$, $Pr=0.1$, and $n=1.2$

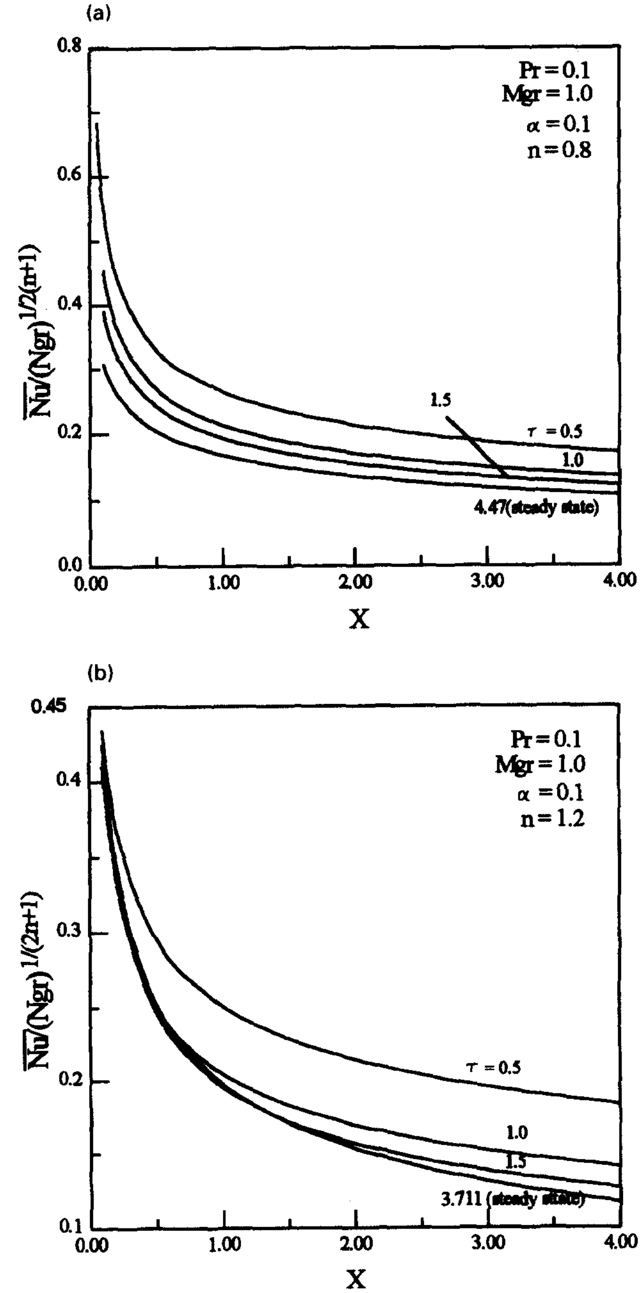


Figure 6 (a) Transient average heat transfer coefficient distribution for $\alpha=0.1$, $Pr=0.1$, and $n=0.8$; (b) transient average heat transfer coefficient distribution for $\alpha=0.1$, $Pr=0.1$, and $n=1.2$

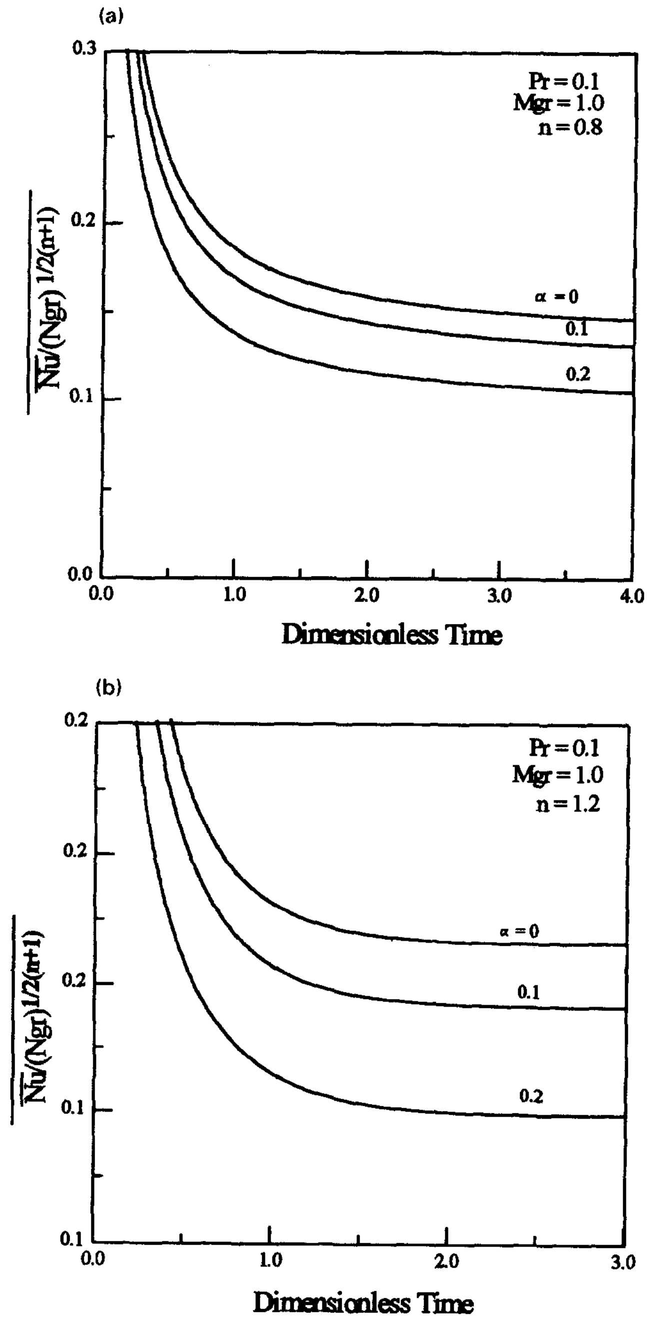


Figure 7 (a) Average Nusselt number distribution for different wave amplitude Pr=0.1 and n=0.8; (b) average Nusselt number distribution for different wave amplitude Pr=0.1 and n=1.2

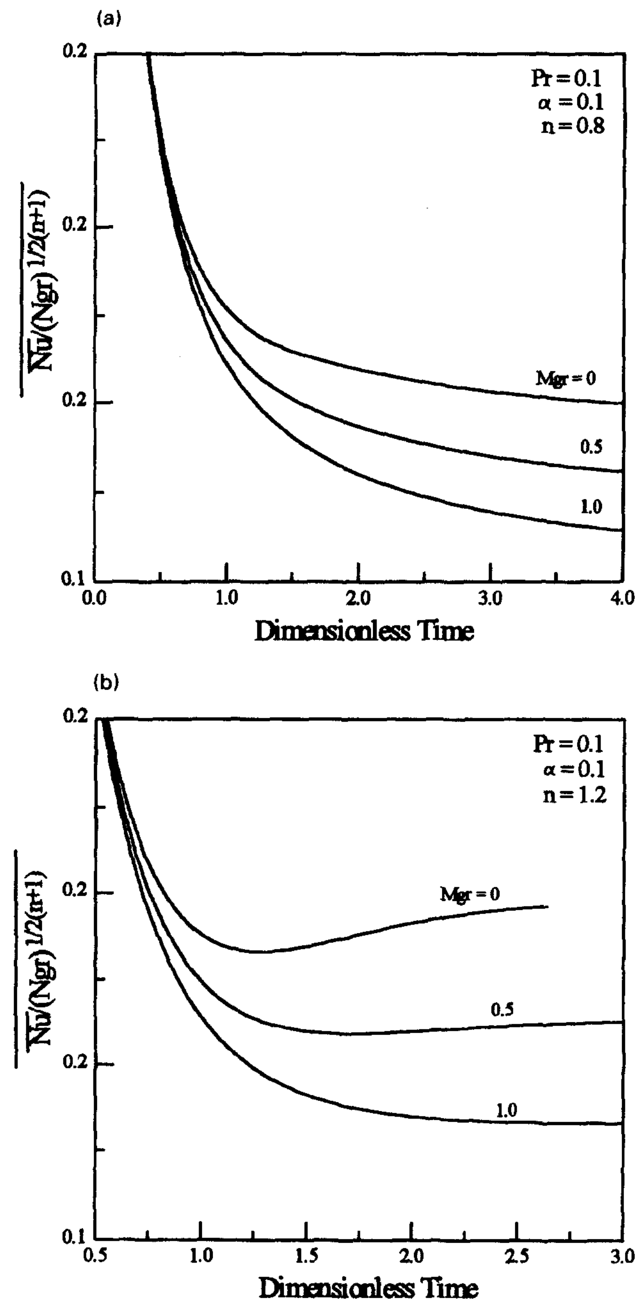


Figure 8 (a) Average Nusselt number distribution for different magnetic strength, Pr=0.1 and n=0.8; (b) average Nusselt number distribution for different magnetic strength Pr=0.1 and n=1.2

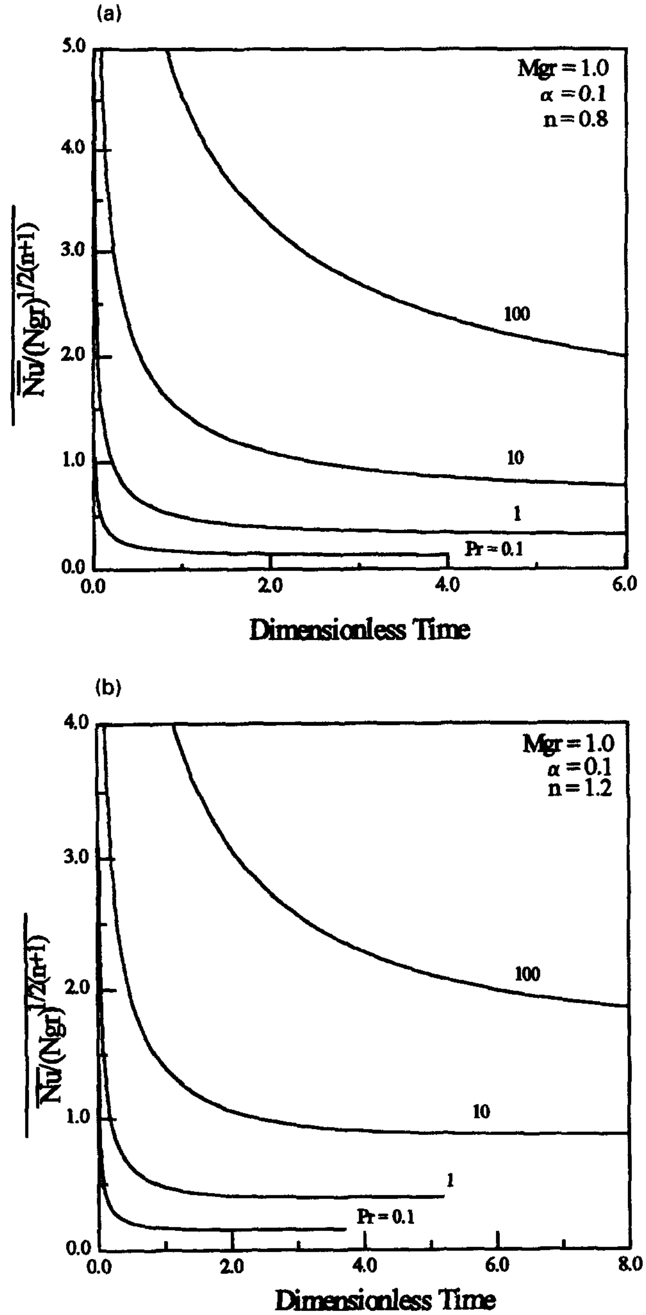


Figure 9 (a) Average Nusselt number distribution for different Prandtl number $n=0.8$; (b) average Nusselt number distribution for different Prandtl number $n=1.2$

Conclusions

A Prandtl transformation method is applied to study the transient free convection of a non-Newtonian fluid along a wavy surface. A sinusoidal surface $\delta = \alpha \sin(2\pi x)$ is used to demonstrate the advantages of the transformation method and to present the heat transfer mechanism near such surfaces. It is also shown that the magnetic field can be used to control the flow characteristics.

References

Acrivos, A. 1960. A theoretical analysis of laminar natural convection heat transfer to non-Newtonian fluids. *AIChE J.* **6**, 584–590.

Chen, J. L. S. 1974. Natural convection of power-law fluids from a vertical plate with uniform surface heat flux. *Proc. 5th Int. Heat Transfer Conference*, Science Council of Japan, Vol. **3**, 39–43

Haq, S., Kleinstreuer, C. and Mulligan, J. C. 1988. Transient free convection of a non-Newtonian fluid along a vertical wall. *J. Heat Transfer*, **110**, 604–607

Huang, M., Huang, J., Chou, Y. and Chen, Y. 1989. Effects of Prandtl number on free convection heat transfer from a vertical plate to a non-Newtonian fluid. *J. Heat Transfer*, **111**, 189–191

Kawase, Y. and Ulbrecht, J. J. 1984. Approximate solution to the natural convection heat transfer from a vertical plate. *Int. Comm. Heat Mass Transfer*, **11**, 143–155

Kim, E. and Chen, J. L. S. 1991. Natural convection of non-Newtonian fluids along a wavy vertical plate. In *Fundamentals of Heat Transfer in Non-Newtonian Fluids*, HTD, Vol. **174**, ASME, New York, 45–49

Kleinstreuer, C., Wang, T. Y. and Haw, S. 1987. Natural convection heat transfer between a power-law fluid and a permeable isothermal vertical wall. In *Fundamentals of Convection in Non-Newtonian Fluids*, J. L. S. Chen, J. M. Ekmann and G. P. Peterson, eds. HTD, Vol. **79**, ASME, New York, 25–31

Lee, S. Y., and Ames, W. F. 1966. Similarity solutions for non-Newtonian fluids. *AIChE*, **12**, 700–708

Na, T. Y. and Hansen, A. G. 1966. Possible similarity solutions of the laminar natural convection flow of non-Newtonian fluids. *Int. J. Heat Mass Transfer*, **9**, 261–262

Nanbu, K. 1971. Limit of pure conduction for unsteady free convection on a vertical plate. *Int. J. Heat Mass Transfer*, **14**, 1531–1534

Rubin, S. G. and Graves, R. A. 1975. Viscous flow solution with a cubic spline approximation. *Computers Fluids*, **3**, 1–36

Shenoy, A. V. and Mashelkar, R. A. 1982. Thermal convection in non-Newtonian fluids. *Adv. Heat Transfer*, **15**, 143–225.

Som, A. and Chen, J. L. S. 1984. Free convection of nonisothermal two-dimensional bodies. *Int. J. Heat Mass Transfer*, **27**, 791–794

Williams, J. C., Mulligan, J. C., and Rhyne, T. B. 1987. Semi-similar solutions for unsteady free-convective boundary layer flow over a vertical flate plate. *J. Fluid Mech.*, **175**, 309–332

Yao, L. S. 1983. Natural convection along a vertical wavy surface.